Scaling of the Hall Resistivity in the Solid and Liquid Vortex Phases in Twinned Single Crystal YBa$_2$Cu$_3$O$_{7-\delta}$

G. D'Anna,[1] V. Berseth,[1] L. Forró,[1] A. Erb,[2] and E. Walker[2]

1 Institut de Génie Atomique, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
2 Département de Physique de la Matière Condensée, Université de Genève, CH-1211 Genève, Switzerland

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Longitudinal and Hall voltages are measured in a clean twinned $YBa_2Cu_3O_{7-\delta}$ single crystal in the liquid and solid vortex phases. For magnetic fields tilted away from the c-axis more than about $2°$, a scaling law $|\rho_{xy}| = A\rho_{xx}^{\beta}$ with $\beta \approx 1.4$ is observed, which is unaffected by the vortex-lattice melting transition. The vortex-solid Hall conductivity is non-linear and diverges to negative values at low temperature. When the magnetic field is aligned to the c-axis, the twin-boundary correlated disorder modifies the scaling law, and $\beta \approx 2$. The scaling law is unaffected by the Bose-glass transition. We discuss the scaling behaviour in terms of the dimension-dependent theory for percolation in metallic conductors.

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A current flowing in a conductor exposed to a magnetic field gives rise to a Hall voltage. The Hall effect has been a powerful probe of the mechanisms of charge transport in metals and semiconductors. Similarly, a Hall voltage is observed in superconductors in high magnetic fields and carrying large electric currents. The Hall effect in this system is an intriguing phenomenon which has triggered a very large experimental and theoretical literature. Remarkable experimental facts include the "Hall anomaly", i.e., the Hall effect sign reversal in the superconducting vortex state with respect to the normal state, as observed in various high- and low-temperature type-II superconductors [1], and the "scaling law", i.e., the power-law dependence of the Hall resistivity with respect to the longitudinal resistivity [2].

Many theoretical explanations have been proposed, most of them addressing the Hall anomaly which is belived to be a fundamental problem of vortex dynamics. These theories are developed either in terms of microscopic electronic processes [3] [4] [5] [6] [7], or including pinning [8], vortex-vortex interactions [9] [10], time-dependent Ginzburg-Landau theories [11], phenomenological models [12] [13] [14], or other ideas [15]. The most frequently adopted approach is microscopic: it ascribes the Hall effect in the vortex state to hydrodynamic and vortex-core forces which determine the single vortex trajectory (e.g. in ref. [4]). In the scenario the Hall sign reversal results from microscopic details of the Fermi surface. The situation remains, however, debated and a consensus is not achieved on fundamental points like the transverse force on a vortex moving in a superfluid [16] [17], or on experimental problems like the doping dependence [18].

Recently the vortex-lattice melting transition has been shown to influence the Hall conductivity [19], which has rised questions to which extent the microscopic approach of the Hall behavior in the vortex state is legitimate [20]. We report here new measurements intended to study the scaling law as the system crosses the vortex-lattice melting transition, or the Bose-glass transition when twin-boundary correlated disorder is relevant. Testing the scaling law in different vortex phases provides insights into the origin of the Hall effect and the mechanisms of magnetic flux transport in type-II superconductors.

The experiments are performed in a very clean twinned $YBa_2Cu_3O_{7-\delta}$ (YBCO) single crystal in which the characteristic features associated to the vortex phase transitions are observed. The micro-twinned crystal has dimensions $0.9 \times 0.4mm^2$ in the a-b plane, and thickness $24\mu m$ in the c-direction. The major twin family is at $45°$ from the long edge of the sample. Some untwinned domains and some twins at $90°$ from the dominant family are also present. The sample displays a sharp resistive transition at about $T_c = 93.5K$. The longitudinal resistivity ρ_{xx} and Hall resistivity ρ_{xy} are measured simultaneously by injecting an ac current $(30Hz)$, sometimes on the top of a dc current, along the longest dimension of the crystal, and by measuring the in-phase voltages parallel and perpendicular to the current. The experimental method is presented in detail in a previous work on the same sample [19]. The Hall conductivity is obtained by $\sigma_{xy} = \rho_{xy}/\left(\rho_{xx}^2 + \rho_{xy}^2\right)$, and the Hall angle θ_H by $\tan\theta_H = \rho_{xy}/\rho_{xx}$.

We begin by discussing the angular dependence and the effect of twin-boundaries in our sample. The Bose-glass theory [21] predicts that for magnetic fields well aligned to the twin-boundaries the vortex-solid phase is a smecticlike phase and the transition to the vortex-liquid is a Bose-glass transition. When the field is tilted away from the twin-boundaries the vortex-solid phase is a Bragg-glass [22] and the transition to the vortex-liquid is a vortex-lattice melting transition.

We found experimentally evidence for this angular behavior. Figure 1 shows the longitudinal resistivity ρ_{xx} measured at $6T$ for zero dc current and a small ac current of $j_{ac} = 1A/cm^2$, and for different angles α between the applied magnetic field and the c-axis, as a function of the temperature. One can clearly see the effect of

twin-boundaries below T_{TB} . The twin-boundary pinning reduces the longitudinal resistivity, as expected for correlated disorder [21]. The inset of Fig. 1 shows the onset temperature T_{onset} of ρ_{xx} measured with a criterion of $0.1\mu\Omega cm$, as a function of the angle. For decreasing, large angles $T_{onset} (= T_m)$ decreases according to a usual anisotropy law [23]. For about $\alpha < 2°$, the onset temperature $T_{onset} (= T_{BG})$ increases and reaches a maximum at $\alpha = 0°$. This kind of behavior has been associated [24] to the change in the nature of the transition according to the Bose-glass theory and the crossover angle is about $2°$ for twinned YBCO crystals similar to the one we use. Therefore, the onset in resistivity in Fig. 1 can be associated to the vortex-lattice melting transition [25] [26] for $\alpha > 2°$, that we denote by T_m, and to the Bose-glass transition [27] for $\alpha < 2°$, that we denote by T_{BG}.

When the vortex-lattice response is probed by superimposing the ac current on top of a large dc current, a longitudinal resistivity different from zero is observed below T_m or T_{BG}, as the vortex-solid is moving under the effect of the large Lorentz force. We then also detect a Hall voltage, and obtain the Hall resistivity and the Hall conductivity in the vortex-solid phase [19]. The inset of Fig. 2 shows the Hall conductivity σ_{xy} as a function of the temperature at $2T$ and at $\alpha = 7°$ and $\alpha = 0°$, measured with large dc and ac current densities ($j_{dc} = 150A/cm^2$, $j_{ac} = 50A/cm^2$) so that the Hall signal is detected deep inside the vortex-solid. In the inset the small difference in temperature between the vortex-lattice melting transition at T_m for $\alpha = 7°$ and the Bose-glass transition at T_{BG} for $\alpha = 0°$ is not visible. By reducing the temperature from the normal state the Hall conductivity σ_{xy} becomes negative below T_c. In the vortex-liquid phase the Hall conductivities at $\alpha = 0°$ and $\alpha = 7°$ coincide down to about T_{TB}. Below roughly T_{TB} and for $\alpha = 0°$ we observe an approximately constant Hall conductivity until the large scattering of the data begins. For the angle tilted away from the c-axis, $\alpha = 7°$, the Hall conductivity decreases smoothly until the vortex-lattice melting transition. Below T_m the Hall conductivity deviates from its behavior in the vortex-liquid phase and goes rapidly towards large negative values (see also the current dependence in Fig. 3 below). The Hall angle, not shown in Fig. 2, tends to small values.

We investigate now the scaling behavior between the Hall resistivity and the longitudinal resistivity, that is the existence of a scaling law $\rho_{xy} \propto \rho_{xx}^\beta$. The main panel of Fig. 2 shows the log-log plot of $|\rho_{xy}|$ vs ρ_{xx} for $\alpha = 7°$ and $\alpha = 0°$ at $2T$ and large dc and ac current densities. The position of the vortex-lattice melting and Bose-glass temperatures are indicated. The fit to a power-law dependence of a form $|\rho_{xy}| = A\rho_{xx}^\beta$, gives for $\alpha = 0°$ the values $A \approx 0.005$ and $\beta \approx 2.0$, and for $\alpha = 7°$ it gives $A \approx 0.02$ and $\beta \approx 1.4$, as shown by the two straight dotted lines. A separate fit to the solid and liquid part gives the same result within the experimental error (we

also obtain $\beta \approx 1.4$ in the whole range $3°$ to $7°$). *There is no change of the $|\rho_{xy}|$ vs ρ_{xx} dependence at the vortex-lattice melting transition or at the Bose-glass transition*, suggesting that such a scaling law is effectively insensitive to the specific vortex phase.

The Hall effect current dependence is shown in Fig. 3 for $\alpha = 3°$. The inset of Fig. 3 shows the Hall conductivity σ_{xy} as a function of the magnetic field at $89K$ and different dc currents. At $\alpha = 3°$ the vortex phase transition is the vortex-lattice melting at B_m. The curves have larger noise over signal ratio than above. Nevertheless the current dependence is clearly observable in the diverging σ_{xy}. *Below the melting field B_m the Hall conductivity σ_{xy} decreases faster, the smaller the dc current.* Above B_m in the vortex-liquid the Hall conductivity is linear. The current dependence of the scaling law is investigated in the main panel of Fig. 3, which shows a log-log plot of $|\rho_{xy}|$ vs ρ_{xx}, constructed from measurements as a function of the magnetic fields at a constant temperature of $89K$ and different current densities. The fit to a power-law dependence of a form $|\rho_{xy}| = A\rho_{xx}^\beta$, with A and β free parameters as above, gives the average values $A \approx 0.012$ and $\beta \approx 1.4$. *There is no change of the scaling law with the current density*, neither in liquid nor in the solid vortex phases.

The data presented here prove that the general trend of the Hall conductivity is indeed captured by a very robust scaling law $|\rho_{xy}| = A\rho_{xx}^\beta$. The scaling law implies $\tan\theta_H \propto \rho_{xx}^{\beta-1}$ and $\sigma_{xy} \propto \rho_{xx}^{\beta-2}$. Consistently, with an exponent less than two at $\alpha > 2°$, as the longitudinal resistivity tends to zero, the Hall conductivity diverges, and the Hall angle is small. The strong non-linear dependence of the longitudinal resistivity in the vortex-solid phase is reflected in the Hall conductivity, which below the melting transition diverges faster, the smaller the current density. For $\alpha = 0°$ and consistently with an exponent $\beta \approx 2$ the Hall conductivity seems to be a constant below about T_{TB}, the temperature of twin-boundary pinning onset.

A Hall resistivity which vanishes as a power of the longitudinal resistivity, $\rho_{xy} \propto \rho_{xx}^\beta$, has been observed by various authors [2] [28] [29], and predicted in different theoretical contexts. Dorsey et al. [13] developed a scaling theory near the vortex-glass transition with a power $\beta < 2$ for the three-dimensional regime. In the model the exponent β is universal, but the sign of the Hall effect is material specific and possibly related to microscopic pairing processes. Vinokur et al. [12] proposed that the scaling law with $\beta = 2$ is a general feature of any vortex state with disorder-dominated dynamics, without the need to invoke the vortex-glass scaling. In this model the Hall conductivity is independent of disorder and is directly linked to the microscopic processes determining the single vortex equation of motion. Wang et al. [8] have proposed that the pinning affects the single vortex trajectory via the backflow current inside the normal vortex-core. This modifies the exponent in such a way that $\beta = 1.5$

for strong pinning and $\beta = 2$ for weak pinning, and the Hall sign reversal is a pinning effect. Ao [9] derived the scaling law in the context of a vortex many-body linear theory, where the Hall voltage results from the motion of vortex-lattice defects (vacancies), with $\beta = 2$ for pinning induced vacancies and $\beta = 1$ for thermally (fluctuation) induced vacancies.

A complete explanation of the puzzling vortex-Hall behavior is not yet achieved. A consistent theory should explain, in addition to the Hall effect sign reversal, the robustness of the scaling $\rho_{xy} \propto \rho_{xx}^\beta$ reported here. We have found that σ_{xy} becomes current dependent in the vortex-solid phase. This contradicts the idea that the Hall conductivity is independent of disorder [12]. *Pinning effects have to be considered.* However, even if the temperature and field dependence of σ_{xy} change at vortex phase transitions (either vortex-lattice melting or Bose-glass transitions), the scaling law is found to remain independent of the specific vortex phase. This suggests that a comprehensive theory for the Hall effect far enough from the sign change does not require phase-dependent parameters. Provided it reproduces a general scaling law $\rho_{xy} \propto \rho_{xx}^\beta$ and leads to the correct sign of σ_{xy}, the Hall behavior is then completely determined by the longitudinal resistivity, which englobes many-body effects, e.g., collective pinning in the vortex-solid phase.

A significant result of this paper is that the exponent β entering the scaling law is *disorder-type dependent.* In particular $\beta \approx 2.0$ for correlated planar disorder and $\beta \approx 1.4$ for uncorrelated point disorder. This suggests an alternative explanation for the scaling behaviour, as proposed by Geshkenbein [30]. If one views the vortex freezing as an inhomogeneous, non simultaneous process, with regions where vortices are pinned (thus with vanishing resistivity), and regions where they can still move (thus inducing a non zero electric resistivity), the vortex freezing behaviour in superconductors has strong analogies with the *percolation transition* in inhomogeneous conductors. In the case of a mixed metallic/insulating system, the conductivity is governed by percolation processes and the longitudinal conducticity is expressed as $\sigma_{xx} \propto \delta p^t$, where $\delta p = p - p_c$ is the difference between the conducting metallic phase density p and the critical percolation threshold density p_c. The critical exponent is $t \approx 1.3$ in two dimensions, and $t \approx 1.6$ in three dimensions [31]. Similarly, the Hall number R_H diverges as $R_H \propto \delta p^{-g}$, where $g = \nu(d-2)$, [31] where d is the dimension. In two dimensions $g = 0$ and as consequence the Hall conductivity $\sigma_{xy} \approx HR_H\sigma_{xx}^2$ is exactly proportional to σ_{xx}^2 (at constant magnetic field H). In three dimensions, $g = \nu \approx 0.9$, [31] such that $\sigma_{xy} \propto \sigma_{xx}^{2-g/t} \propto \sigma_{xx}^{1.44}$.

This immediately leads to a percolation model for the vortex scaling behaviour, provided the vortex conductivity is interpreted as the electric resistivity of the metallic/insulating system, since a high vortex mobility means large electric dissipation. With the identification that the conductivity σ in the metallic/insulating system is the resistivity ρ in the vortex system, one obtains the scaling law $\rho_{xy} \propto \rho_{xx}^{2-g/t}$ and $\beta = 2 - g/t$ for the vortex system. The percolation model is very appealing since it provides two universal exponents, i.e., $\beta = 2$ and $\beta = 1.44$, which correspond to the most frequently reported experimental estimate. These exponents are determined by the dimensionality of the vortex system, that is, determined by the intrinsic anisotropy of the material, or by the vortex localization along correlated defects, or by the geometry of the samples. For the magnetic field accurately aligned to the twin-boundaries, which localizes the vortices along the c-axis, the system is two dimensional and $\beta \approx 2$. The same exponent is observed in two-dimensional films [29]. When the magnetic field is "slightly" tilted away from the twin-boundaries (2 to 3 degrees are enough), the vortices recover the third degree of freedom and $\beta \approx 1.4$. This is also likely to happen when splayed defects are introduced by irradiation, bringing back the exponent form 2 to 1.5. [28]

The vortex-percolation model is indeed very interesting since it predicts a scaling law independent of the vortex phase, as observed here, and explains the robustness of the scaling since it is related only to the dimensionality of the vortex system.

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Figure captions

FIG. 1. The longitudinal resisitivity ρ_{xx} in a twinned $YBa_2Cu_3O_{7-\delta}$ single crystal at $6T$ as a function of the temperature, measured at low ac current density, $j_{ac} = 1A/cm^2$, for different angles between the field and the c-axis (-0.2°, 0°, 0.2°, 0.5°, 1°, 2°, 3°, 5°, 7°, 10°, 12°, 20°). Inset: the onset temperature T_{onst} as a function of the angle, at $2T$ and $6T$. For about $\alpha > 2°$ the onset temperature follows the usual anisotropic law of the vortex-lattice melting temperature. For small angles the onset temperature increases as expected from the Bose-glass theory (see text for details).

FIG. 2. Log-log plot of $|\rho_{xy}|$ versus ρ_{xx} at $2T$ and $\alpha = 7°$ and $\alpha = 0°$. The linear fit according to a scaling law $|\rho_{xy}| = A\rho_{xx}^{\beta}$ gives $\beta \approx 1.4$ for $\alpha = 7°$, and $\beta \approx 2.0$ for $\alpha = 0°$, as indicated by the two straigth dotted lines. The position of the vortex-lattice melting temperature, T_m, and of the Bose-glass temperature, T_{BG}, are indicated in the curves, as well as the temperature of twin-boundary pinning onset, T_{TB}. Notice that the scaling law is unaffected by crossing the transitions. Inset: the Hall conductivity, $\sigma_{xy} = \rho_{xy}/\left(\rho_{xx}^2 + \rho_{xy}^2\right)$, as a function of the temperature at $\alpha = 7°$ and $\alpha = 0°$. The dotted vertical lines denote the transitions at T_m, T_{BG} and T_{TB}. The current densities are $j_{dc} = 150A/cm^2$, $j_{ac} = 50A/cm^2$.

FIG. 3. Log-log plot of $|\rho_{xy}|$ versus ρ_{xx} for different current densities at $89K$ and $\alpha = 3°$. For each curve the current densities are indicated by (j_{dc}, j_{ac}), both in unit of A/cm^2. The position of the vortex-lattice melting field is indicated. The linear fit according to a scaling law $|\rho_{xy}| = A\rho_{xx}^{\beta}$ gives the average values $A \approx 0.012$ and $\beta \approx 1.4$ (see dotted line) and there is no current dependence of the parameters. Inset: The field dependence of the Hall conductivity σ_{xy} at $89K$, for various ac and dc current densities. The dotted vertical line denotes the vortex-lattice melting transition at B_m.